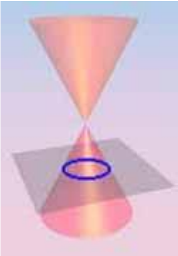
Conic Functions

A conic section is the intersection of a plane and a double-napped cone. Four basic conics are shaped this way as shown in the figures below. Also, notice that the intersecting plane does not pass through the vertex of the cone. Conics is defined as intersecting planes given algebraically in terms of a general second-degree equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0.$$

Circle: A = C; Parabola: AC = 0, both are not zero; Ellipse: AC > 0 Hyperbola: AC < 0



Circle

$(x – h)^2 + (y – k)^2 = r^2$

Center: (h,k)

Radius: r

Conic form: $x^2 + y^2 + Dx + Ey + F = 0$ (notice that A and C equal 1)

To put into standard form of a circle you must use completing the square method, making sure you add what was needed to complete the square to the other side of the equal sign.

Given the center of the circle at (6,-2) and a radius of 4, we want to find the general form of the circle and then in the conic form.

General form: $(x – h)^2 + (y – k)^2 = r^2$ Center: (6,-2) h = 6, k = (-2) and r = 4

$(x – 6)^2 + (y – [-2])^2 = 4^2$

$(x – 6)^2 + (y + 2)^2 = 16$ Expand to get the conic form.

$x^2 + (-12x) + \mathbf{36} + y^2 + 4y + \mathbf{4} = 16$ Add like terms and rearrange into conic format.

$x^2 + y^2 + (-12x) + 4y + \mathbf{40} \mathbf{+ (-16)} = 16 \mathbf{+ (-16)}$ Add (-16) to both sides.

Conic form: $x^2 + y^2 + (-12x) + 4y + 24 = 0$

We can go from the conic form of the equation of a circle to get its general form. From the general form we can get the center of the circle and the radius. Let's use the conic form above to get the general form.

Conic Form: $x^2 + y^2 + (-12x) + 4y + 24 = 0$ Bring the x terms together and the y terms together.

$x^2 + (-12x) + y^2 + 4y + 24 \mathbf{+ (-24)} = 0 \mathbf{+ (-24)}$ Add (-24) to both sides and leave space

$x^2 + (-12x) + \quad y^2 + 4y + \quad = (-24)$ to complete the square.

$x^2 + (-12x) \mathbf{+ (-6)^2}$ $y^2 + 4y \mathbf{+ (2)^2}$

$\mathbf{(x – 6)^2}$ $\mathbf{(y + 2)^2}$

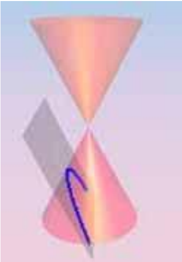
$x^2 + (-12x) \mathbf{+ (-6)^2} + y^2 + 4y \mathbf{+ (2)^2} = (-24) \mathbf{+ (-6)^2} \mathbf{+ (2)^2}$

$\mathbf{(x – 6)^2} + \mathbf{(y + 2)^2} = (-24) + 36 + 4$

General form: $(x – 6)^2 + (y + 2)^2 = 16$

h = 6, k = (-2) and $r = \sqrt{16} = 4$

Center: (6, -2) and r = 4



Parabola	Vertical Axis	Horizontal Axis
Standard form	$(x-h)^2 = 4p(y-k)$	$(y-k)^2 = 4p(x-h)$
Opens	Up $p > 0$ or Down $p < 0$	Left $p < 0$ or Right $p > 0$
Vertex	(h, k)	(h, k)
Focus	(h, k + p)	(h + p, k)
Directrix	$y = k - p$	$x = h - p$
Axis of symmetry	$x = h$	$y = k$
Focal length	p	p
Focal width (diameter)	$\|4p\|$	$\|4p\|$
Conic form	$x^2 + Dx + Ey + F = 0$	$y^2 + Ey + Dx + F = 0$

From the standard form of a parabola we should be able to determine the vertex, focal length, focus and the directrix. Let's look at an example.

$-8(x-2) = y^2$

$(y-0)^2 = (-8)(x-2)$ from the standard form $(y-k)^2 = 4p(x-h)$ h = 2 and k = 0 with 4p = (-8)

Vertex: (2,0)

Focal length (p): (-2) {divided 4 to both sides of 4p = 8} {p < 0 opening to the left}

Focus: (h + p,k) → (2 + (-2),0) → (0,0)

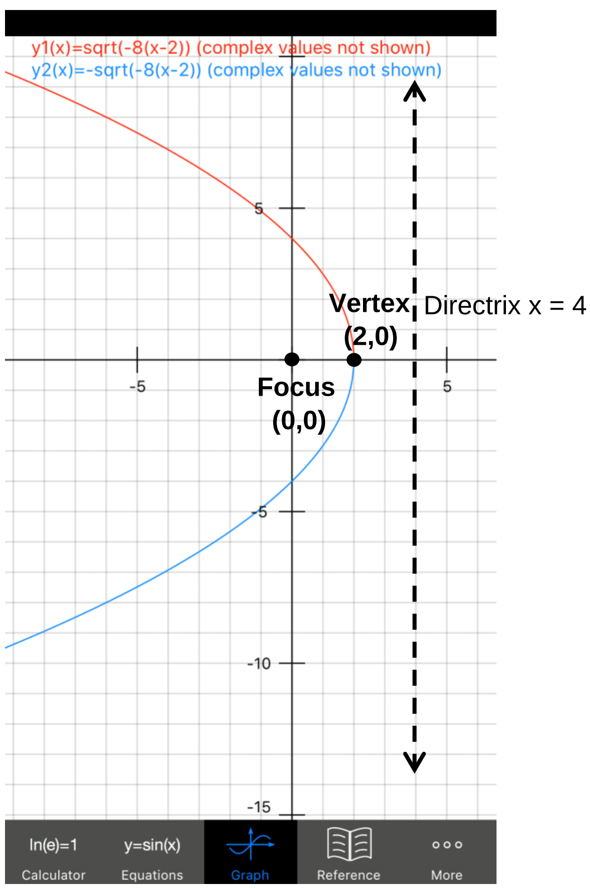
Directrix: $x = h - p$ → $x = 2 - (-2) = 2 + 2 = 4$ $x = 4$

To put $-8(x-2) = y^2$ into conic form by rearranging the equation.

$(-8x) + 16 = y^2$

$(-y^2) + (-8x) + 16 = 0$

$y^2 + 8x + (-16) = 0$



Find the general equation for a parabola from the conic form and find the vertex, focal length, focus and the directrix.

$x = 2y^2 – 8y + 24 \rightarrow x = 2y^2 + (-8y) + 24$

$x = 2[y^2 + (-4y) + 12]$

$x = 2[y^2 + (-4y) + (-2)^2 + (-4) + 12]$ Complete the square and add the opposite.

$x = 2[(y - 2)^2 + 8]$

$\frac{1}{2}x = (y - 2)^2 + 8$

$\frac{1}{2}x + (-8) = (y - 2)^2$ Factor 1/2 .

$\frac{1}{2}[x + (-16)] = (y - 2)^2$ $(y – k)^2 = 4p(x – h)$ h = 16, k = 2 and $4p = \frac{1}{2}$

Vertex: (16, 2)

Focal length (p): $p = \frac{1}{8}$ p > 0 opens right

Focus: $(h + p,k) \rightarrow (16 + \frac{1}{8}, 2) \rightarrow (16\frac{1}{8}, 2)$

Directrix: $x = h – p \rightarrow x = 16 – \frac{1}{8} \rightarrow x = 15\frac{7}{8}$

Given the vertex (3,2) and the focus $(3, \frac{5}{2})$ of a parabola, find the general equation and conic equation.

Notice that the x = 3 so x = h = 3 (vertical axis) and k = 2.

Focus (h, k + p) $k + p \rightarrow 2 + p = \frac{5}{2}$ $p = \frac{5}{2} - \frac{4}{2} = \frac{1}{2}$

$(x – h)^2 = 4p(y – k)$

$(x – 3)^2 = 4\frac{1}{2}(y – 2)$

$(x – 3)^2 = 2(y – 2)$ **general equation**

$x^2 + (-6x) + 9 = 2y + (-4)$ rearrange the equations

$x^2 + (-6x) + (-2y) + 9 + 4 = 2y + (-4) + (-2y) + 4$

$x^2 + (-6x) + (-2y) + 13 = 0$ **conic equation**

Given the vertex (-1,2) and the focus (3,2) of a parabola, find the general equation and conic equation.

Notice that the y = 2 so y = k = 2 (horizontal axis) and h = (-1).

Focus (h + p,k) $h + p \rightarrow (-1) + p = 3$ $p = 3 + 1$ $p = 4$

$(y – k)^2 = 4p(x – h)$

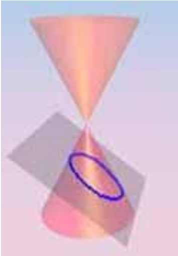
$(y – 2)^2 = 4(4)(x – (-1))$

$(y – 2)^2 = 16(x + 1))$ **general equation**

$y^2 + (-4y) + 4 = 16x + 16$

$y^2 + (-4y) + 4 + (-16x) + (-16) = 0$

$y^2 + (-4y) + (-16x) + (-12) = 0$ **conic equation**



Ellipse	Horizontal Major Axis	Vertical Major Axis
Standard form {a^2 is always largest}	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Focal axis	y = k	x = h
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Co-Vertices	$(h, k \pm b)$	$(h \pm b, k)$
Major axis	2a = length of major axis	2a = length of major axis
Minor axis	2b = length of minor axis	2b = length of minor axis
Semimajor axis	a	a
Semiminor axis	b	b
c (Pythagorean relation)	$a^2 = b^2 + c^2$ or $c^2 = a^2 - b^2$	
Eccentricity of an ellipse	$e = \frac{c}{a}$	
Conic form	$Ax^2 + Cy^2 + Dx + Ey + F = 0$	

Examples:

Find the center, vertices, foci, major axis length, minor axis length and eccentricity of the ellipse. Put into either the standard form or conic form (opposite of what is given). Graph the ellipse.

1) $\frac{(x-1)^2}{4} + \frac{(y-1)^2}{16} = 1$

Rearrange a^2 is the largest denominator.

$$\frac{(y-1)^2}{16} + \frac{(x-1)^2}{4} = 1$$

h = 1 k = 1

$a^2 = 16$ a = 4

$b^2 = 4$ b = 2

$c^2 = a^2 - b^2$

$c^2 = 16 - 4$

$c^2 = 12$

$c = \sqrt{12}$

$c = 2\sqrt{3}$

Center (h, k): (1, 1)

Vertices $(h, k \pm a)$: (1, 1 + 4) → (1, 5)

(1, 1 – 4) → (1, -3)

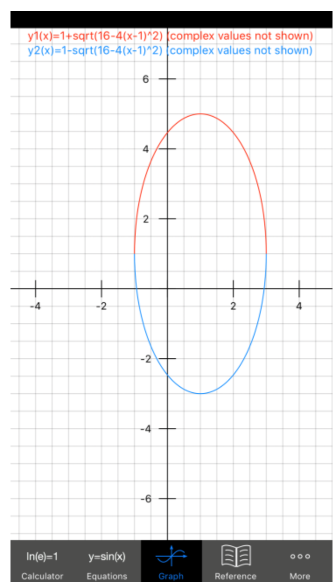
Foci $(h, k \pm c)$: $(1, 1 + 2\sqrt{3})$

$(1, 1 - 2\sqrt{3})$

Major axis length 2a: 8 units

Minor axis length 2b: 4 units

Eccentricity $e = \frac{c}{a}$: $e = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2} \cong 0.866$



$4(y-1)^2 + 16(x-1)^2 = 4(16)$

$4[y^2 + (-2y) + 1] + 16[x^2 + (-2x) + 1] = 64$

$4y^2 + (-8y) + 4 + 16x^2 + (-32x) + 16 = 64$

$16x^2 + 4y^2 + (-32x) + (-8y) + 20 = 64$

$16x^2 + 4y^2 + (-32x) + (-8y) + (-44) = 0$ **conic form**

To graph:

$4(y-1)^2 = 64 - 16(x-1)^2$

$(y-1)^2 = 16 - 4(x-1)^2$

$y - 1 = \pm\sqrt{16 - 4(x-1)^2}$

$y = 1 \pm \sqrt{16 - 4(x-1)^2}$

2) $9x^2 + 16y^2 + (-36x) + (-128y) + 148 = 0$

$9x^2 + (-36x) + \quad 16y^2 + (-128y) + \quad = (-148)$

$9[x^2 + (-4x) + \quad] + 16[y^2 + (-8y) + \quad] = (-148)$

9(4) = 36 16(16) =256

$9[x^2 + (-4x) + (-2)^2] + 16[y^2 + (-8y) + (-4)^2] = (-148)$ **+ 36 + 256**

$9(x – 2)^2 + 16(y – 4)^2 = 144$

$$\frac{(x-2)^2}{16} + \frac{(y-4)^2}{9} = \frac{144}{(16)(9)}$$

$$\frac{(x-2)^2}{16} + \frac{(y-4)^2}{9} = 1$$

$h = 2 \quad k = 4$

$a^2 = 16 \quad a = 4$

$b^2 = 9 \quad b = 3$

$c^2 = a^2 - b^2$

$c^2 = 16 – 9$

$c^2 = 7$

$c = \sqrt{7}$

Center (h, k): (2, 4)

Vertices $(h \pm a, k)$: (2 + 4, 4) → (6, 4)

(2 – 4, 4) → (-2, 4)

Foci $(h \pm c, k)$: $(2 + \sqrt{7}, 4)$

$(2 - \sqrt{7}, 4)$

Major axis length 2a: 8 units

Minor axis length 2b: 6 units

Eccentricity $e = \frac{c}{a}$: $e = \frac{\sqrt{7}}{4} = \frac{\sqrt{7}}{4} \cong 0.661$

To graph:

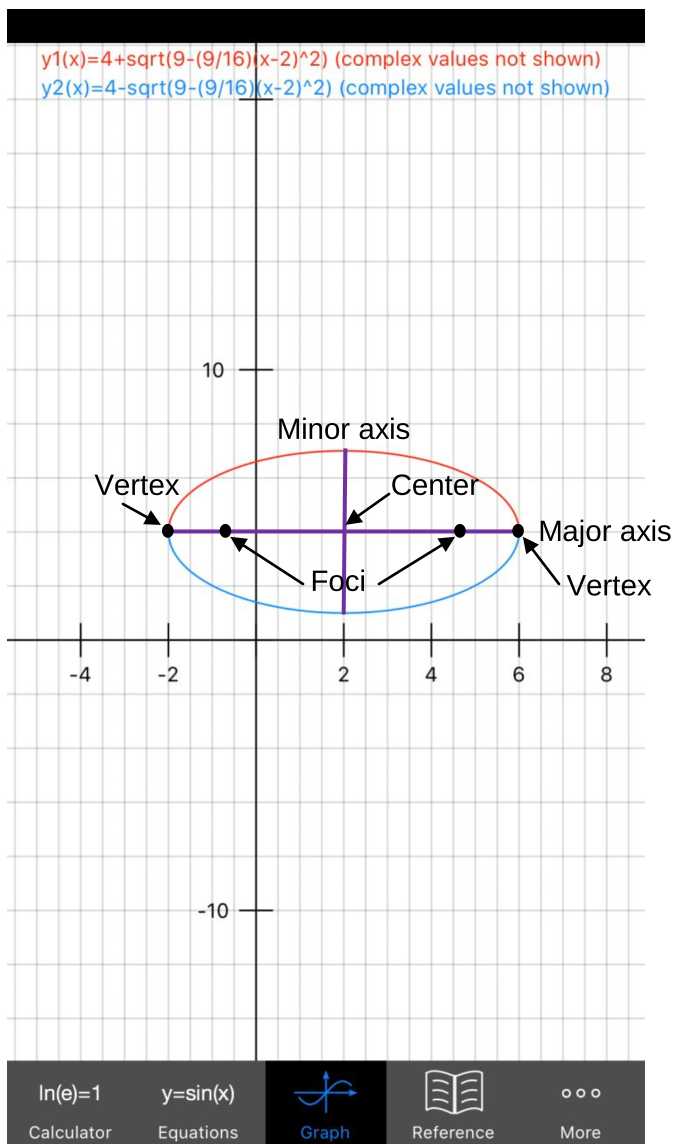
$9(x – 2)^2 + 16(y – 4)^2 = 144$

$16(y – 4)^2 = 144 – 9(x – 2)^2$

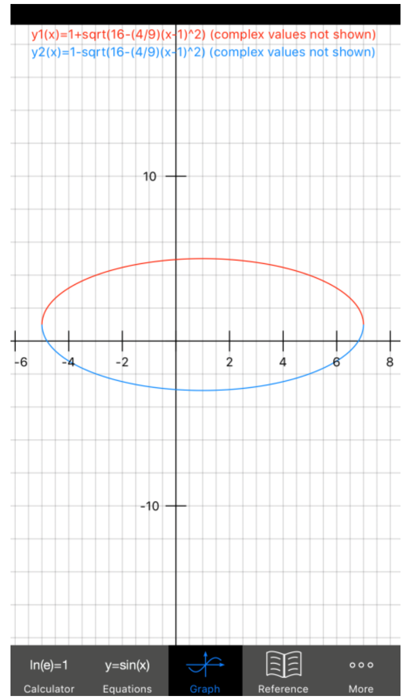
$(y – 4)^2 = 9 - \frac{9}{16}(x – 2)^2$

$$y - 4 = \pm\sqrt{9 - \frac{9}{16}(x-2)^2}$$

$$y = 4 \pm \sqrt{9 - \frac{9}{16}(x-2)^2}$$



3) $\frac{(x+1)^2}{36} + \frac{(y-1)^2}{16} = 1$



$h = (-1)$ $k = 1$

$a^2 = 36$ $a = 6$

$b^2 = 16$ $b = 4$

$c^2 = a^2 - b^2$

$c^2 = 36 - 16$

$c^2 = 20$

$c = \sqrt{20}$

$c = 2\sqrt{5}$

Center (h, k): (-1, 1)

Vertices $(h \pm a, k)$: (5, 1)
(-7, 1)

Foci $(h \pm c, k)$: $(-1 + 2\sqrt{5}, 1)$
$(-1 - 2\sqrt{5}, 1)$

Major axis length 2a: 12 units

Minor axis length 2b: 8 units

Eccentricity $e = \frac{c}{a}$: $e = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} \cong 0.745$

$16(x+1)^2 + 36(y-1)^2 = 36(16)$

$16[x^2 + 2x + 1] + 36[y^2 + (-2y) + 1] = 576$

$16x^2 + (-32x) + 16 + 36y^2 + (-72y) + 36 = 576$

$16x^2 + 36y^2 + (-32x) + (-72y) + 52 = 576$

$16x^2 + 36y^2 + (-32x) + (-72y) + (-524) = 0$ **conic form**

To graph:

$36(y-1)^2 = 576 - 16(x-1)^2$

$(y-1)^2 = 16 - \frac{4}{9}(x-1)^2$

$y - 1 = \pm\sqrt{16 - \frac{4}{9}(x-1)^2}$

$y = 1 \pm \sqrt{16 - \frac{4}{9}(x-1)^2}$

4) $4x^2 + 9y^2 + (-8x) + 36y + (-104) = 0$

$4x^2 + (-8x) + \quad 9y^2 + 36y + \quad = 104$

$4[x^2 + (-2x) + \quad] + 9[y^2 + 4y) + \quad] = (-148)$

4(1) = 4 **9(4) =36**

$4[x^2 + (-2x) + (-1)^2] + 9[y^2 + 4y) + (2)^2] = (-148)$ **+ 4 + 36**

$4(x-1)^2 + 9(y-4)^2 = 144$

$\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = \frac{144}{(16)(4)}$

$\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 4$

$\frac{(x-1)^2}{36} + \frac{(y+2)^2}{16} = 1$

$h = 1$ $k = (-2)$

$a^2 = 36$ $a = 6$

$b^2 = 16$ $b = 4$

$c^2 = a^2 - b^2$

$c^2 = 36 - 16$

$c^2 = 20$

$c = 2\sqrt{5}$

Center (h, k): (1, -2)

Vertices $(h \pm a, k)$: $(1 + 6, -2) \rightarrow (7, -2)$
$(1 - 6, -2) \rightarrow (-5, -2)$

Foci $(h \pm c, k)$: $(1 + 2\sqrt{5}, -2)$
$(1 - 2\sqrt{5}, -2)$

Major axis length 2a: 12 units

Minor axis length 2b: 8 units

Eccentricity $e = \frac{c}{a}$: $e = \frac{2\sqrt{5}}{6} = \frac{\sqrt{5}}{3} \cong 0.745$

To graph:

$9(y+2)^2 = 144 - 4(x-1)^2$

$(y+2)^2 = 16 - \frac{4}{9}(x-1)^2$

$y + 2 = \pm\sqrt{16 - \frac{4}{9}(x-1)^2}$

$y = (-2) \pm \sqrt{16 - \frac{4}{9}(x-1)^2}$

Given the vertices and foci find the general equation and conic equation for the ellipse.

5) Vertices: (16, -3) (-10, -3)
Foci: (8, -3) (-2, -3) k is the same – horizontal axis

Vertex → $(h \pm a, k)$ Foci → $(h \pm c, k)$

h + a = 16 h + c = 8
h – a = (-10) h – c = (-2)
solve for h
add the two equations
2h = 6
h = 3 k = (-3)

find a
3 + a = 16
a = 13
$a^2 = 169$

find c
3 + c = 8
c = 5
$c^2 = 25$

find b
$b^2 = a^2 – c^2$
$b^2 = 169 – 25$
$b^2 = 144$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x - 3)^2}{169} + \frac{(y + 3)^2}{144} = 1$$

$144(x – 3)^2 + 169(y + 3)^2 = (169)(144)$
$144[x^2 + (-3x) + 9] + 169(y^2 + 6y + 9) = 24336$
$144 x^2 + (-432x) + 1296 + 169 y^2 + 1014y + 1521 = 24336$
$144 x^2 + 169 y^2 + (-432x) + 1014y + 2817 = 24336$
$144 x^2 + 169 y^2 + (-432x) + 1014y + (-21519) = 0$

6) Vertices: (5, -6) (-15, -6)
Foci: $(-5 + 5\sqrt{3}, -6)$ $(-5 – 5\sqrt{3}, -6)$ k is the same – horizontal axis

Vertex → $(h \pm a, k)$ Foci → $(h \pm c, k)$

h + a = 5 h + c = 8
h – a = (-15) h – c = (-2)
solve for h
add the two equations
2h = (-10)
h = (-5) k = (-3)

find a
(-5) + a = 5
a = 10
$a^2 = 100$

find c
$c = 5\sqrt{3}$
$c = \sqrt{25(3)}$
$c^2 = \sqrt{75}$

find b
$b^2 = a^2 – c^2$
$b^2 = 1100 – 75$
$b^2 = 25$

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$
$$\frac{(x + 5)^2}{100} + \frac{(y + 6)^2}{25} = 1$$

$25(x + 5)^2 + 100(y + 6)^2 = (100)(25)$
$25[x^2 + 10x + 25] + 100(y^2 + 12y + 36) = 2500$
$25 x^2 + 250x + 625 + 100 y^2 + 1200y + 3600 = 2500$
$25 x^2 + 100 y^2 + 250x + 1200y + 4225 = 2500$
$25 x^2 + 100 y^2 + 250x + 1200y + 1725 = 0$

6) Vertices: (-10, 5) (-10, -9) h is the same – vertical axis
Foci: $(-10, -2 + 2\sqrt{6},)$ $(-10, -2 – 2\sqrt{6},)$

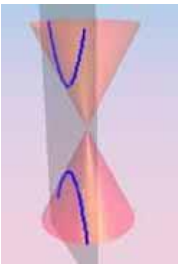
Vertex → $(h, k \pm a)$ Foci → $(h, k \pm c)$

h = (-10) k = (-2) $c = 2\sqrt{6}$
(-2) + a = 5 $c = \sqrt{(4)6}$
a = 7 $a^2 = 49$ $c = \sqrt{24}$
$c^2 = 24$

$b^2 = a^2 – c^2$
$b^2 = 49 – 24$
$b^2 = 25$

$$\frac{(y - k)^2}{a^2} + \frac{(x - h)^2}{b^2} = 1$$
$$\frac{(y + 2)^2}{49} + \frac{(x + 10)^2}{25} = 1$$

$25(y + 2)^2 + 49(x + 10)^2 = (49)(25)$
$25[y^2 + 4x + 4] + 49(y^2 + 20y + 100) = 1225$
$25 y^2 + 100y + 100 + 49 x^2 + 980x + 4900 = 1225$
$49 x^2 + 100 y^2 + 980x + 100y + 5000 = 1225$
$49 x^2 + 100 y^2 + 980x + 100y + 3775 = 0$



Hyperbola

	Horizontal Transverse Axis	Vertical Transverse Axis
Standard form {a^2 is always largest}	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
Center	(h, k)	(h, k)
Focal axis	y = k	x = h
Foci	$(h \pm c, k)$	$(h, k \pm c)$
Vertices	$(h \pm a, k)$	$(h, k \pm a)$
Semitransverse axis	a	a
Semiconjugate axis	b	b
c (Pythagorean relation)	$c^2 = a^2 + b^2$	
Asymptotes	$y = k \pm \frac{b}{a}(x - h)$	$y = k \pm \frac{a}{b}(x - h)$
Conic form	$Ax^2 + Cy^2 + Dx + Ey + F = 0$ note: A or C is negative	

Examples:

$\frac{(y+1)^2}{16} - \frac{(x-1)^2}{9} = 1$ $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

Vertical transverse axis

$h = 1$ $k = (-1)$ $a^2 = 16$ $b^2 = 9$

$a = 4$ $b = 3$

$c^2 = a^2 + b^2$

$c^2 = 16 + 9$

$c^2 = 25$

$c = 5$

center (h, k): (1, -1)

vertices $(h, k \pm a)$: $(1, -1 \pm 4) \rightarrow (1, 3)$

(1, -5)

foci $(h, k \pm c)$: $(1, -1 \pm 5) \rightarrow (1, 4)$

(1, -6)

asymptotes $y = k \pm \frac{a}{b}(x - h)$: $y = (-1) \pm \frac{4}{3}(x - 1)$

$y = (-1) + \frac{4}{3}(x - 1)$ $y = (-1) - \frac{4}{3}(x - 1)$

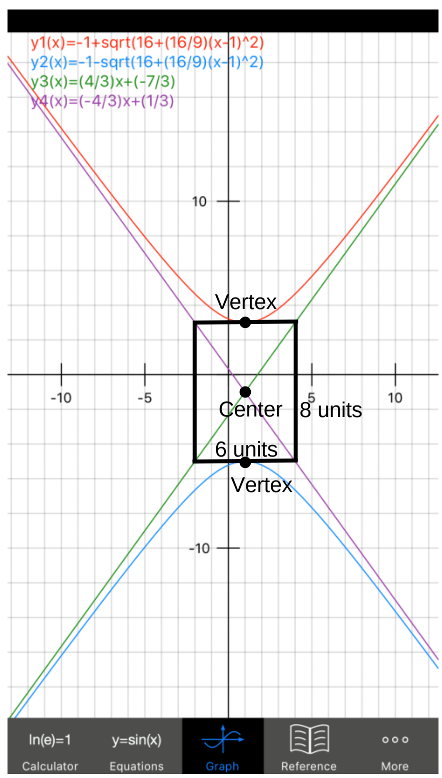
$y = (-1) + \frac{4}{3}x + (\frac{-4}{3})$ $y = (-1) + (\frac{-4}{3}x) + \frac{4}{3}$

$y = (\frac{-3}{3}) + \frac{4}{3}x + (\frac{-4}{3})$ $y = (\frac{-3}{3}) + (\frac{-4}{3}x) + \frac{4}{3}$

$y = \frac{4}{3}x + (\frac{-7}{3})$ $y = (\frac{-4}{3}x) + \frac{1}{3}$

length of transverse axis 2a: 2(4) = 8 units

length of conjugate axis 2b: 2(3) = 6 units



To graph:

1. Plot the vertices.
2. Plot the center.
3. Make a box of 2a by 2b: (a units to the left and a units to the right of the vertices and from the ends of 2a line go up or down 2b units).
4. Draw asymptotes.
5. From vertices draw the hyperbolas.

$$\frac{(y+1)^2}{16} - \frac{(x-1)^2}{9} = 1$$

$9(y + 1)^2 – 16(x – 1)^2 = 9 (16)$ → $9[y^2 + 2y + 1] + (-16)[x^2 + (-2x) + 1] = 144$

$9(y + 1)^2 = 144 + 16(x – 1)^2$

$9y^2 + 18y + 9 + (-16x^2) + 32x + (-16) = 144$

$(y + 1)^2 = 16 + \frac{16}{9}(x – 1)^2$

$(-16x^2) + 9y^2 + 32x + 18y + (-7) = 144$

$y + 1 = \pm\sqrt{16 + \frac{16}{9}(x – 1)^2}$

$(-16x^2) + 9y^2 + 32x + 18y + (-151) = 0$ **conic form**

$y = (-1) \pm\sqrt{16 + \frac{16}{9}(x – 1)^2}$ **graphing form**

Another example:

$$\frac{x^2}{25} - \frac{(y+1)^2}{16} = 1$$

Horizontal transverse axis

$h = 0$ $k = (-1)$ $a^2 = 25$ $b^2 = 16$

$a = 5$ $b = 4$

$c^2 = a^2 + b^2$

$c^2 = 25 + 16$

$c^2 = 41$

$c = \sqrt{41} \cong 6.403$

center (h, k): (0, -1)

vertices $(h \pm a, k)$: $(0 \pm 5, -1)$ → (5, -1)

(-5, -1)

foci $(h \pm c, k)$: $(0 \pm \sqrt{41}, -1)$ → $(\sqrt{41}, -1)$

$(-\sqrt{41}, -1)$

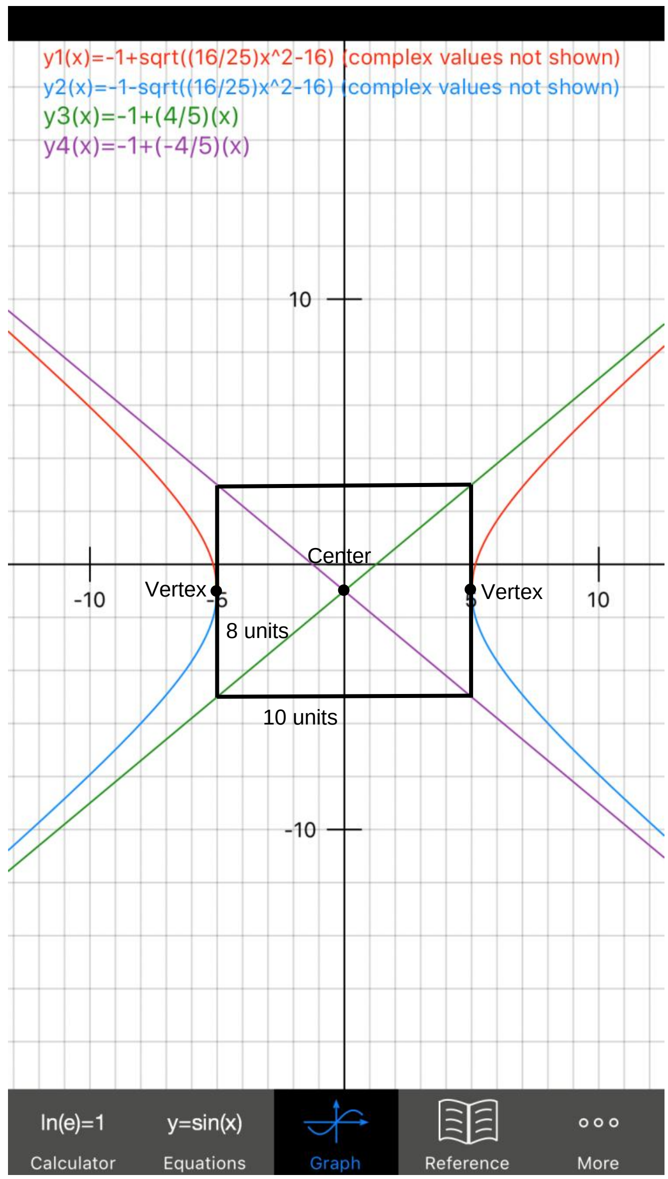
asymptotes $y = k \pm \frac{b}{a}(x - h)$: $y = (-1) \pm \frac{4}{5}(x - 0)$

$y = (-1) \pm \frac{4}{5}x$

$y = (-1) + \frac{4}{5}x$ $\quad$ $y = (-1) - \frac{4}{5}x$

length of transverse axis 2a: 2(5) = 10 units

length of conjugate axis 2b: 2(4) = 8 units



$$\frac{x^2}{25} - \frac{(y+1)^2}{16} = 1$$

$16x^2 – 25(y + 1)^2 = (16)25$

$16x^2 + (-25)(y^2 + 2y + 1) = 400$

$16x^2 + (-25y^2) + (-59y) + (-25) = 400$

$16x^2 + (-25y^2) + (-59y) + (-425) = 0$ **conic form**

$16x^2 – 25(y + 1)^2 = 400$

$16x^2 + (-400) = 25(y + 1)^2$

$\frac{16}{25}x^2 + (-16) = (y + 1)^2$

$\pm\sqrt{\frac{16}{25}x^2 + (-16)} = y + 1$

$(-1) \pm\sqrt{\frac{16}{25}x^2 + (-16)} = y$ **graphing form**

Last example:

$-4x^2 + 9y^2 + 18y + (-27) = 0$

$9y^2 + 18y + -4x^2 = 27$

$9(y^2 + 2y + \quad) - 4x^2 = 27$ complete the square

$9(y^2 + 2y + 1^2) - 4x^2 = 27 + 9$

$9(y + 1)^2 - 4x^2 = 36$

$$\frac{9(y+1)^2}{36} - \frac{4x^2}{36} = \frac{36}{36}$$

$$\frac{(y+1)^2}{4} - \frac{x^2}{9} = 1$$

vertical transverse axis

$h = 0$ $k = (-1)$ $a^2 = 4$ $b^2 = 9$

$a = 2$ $b = 3$

$c^2 = a^2 + b^2$

$c^2 = 4 + 9$

$c^2 = 13$

$c = \sqrt{13} \cong 3.606$

center (h, k): (0, -1)

vertices $(h, k \pm a)$: $(0, -1 \pm 2) \rightarrow (0, 1)$

$(0, -3)$

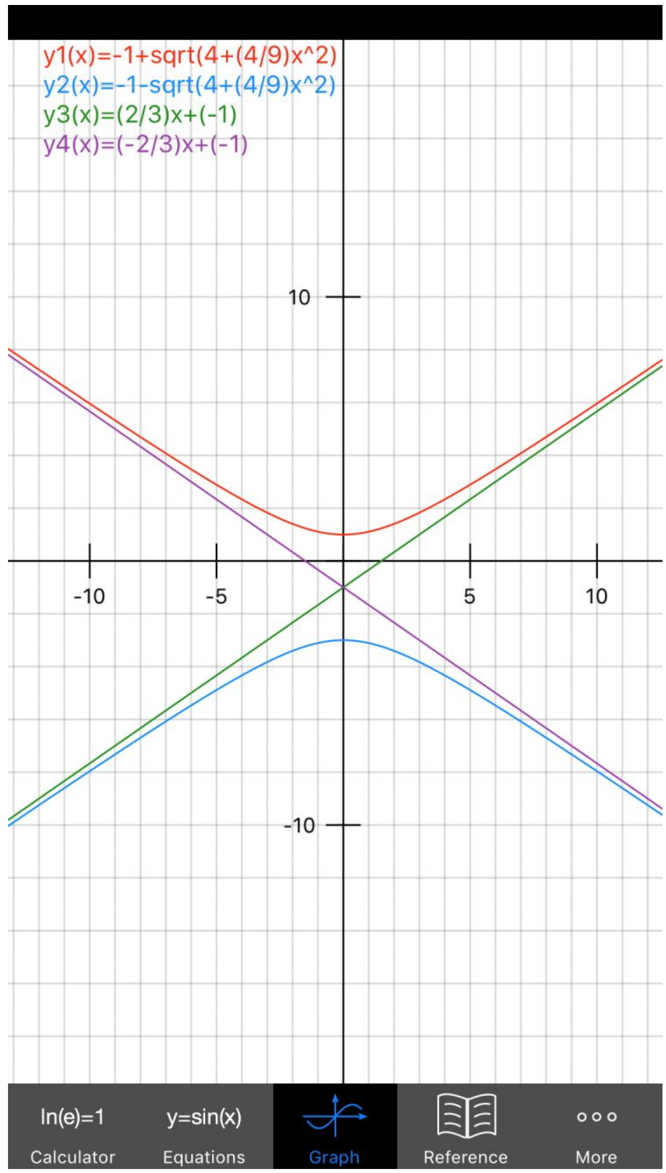
foci $(h, k \pm c)$: $(0, -1 \pm \sqrt{13}) \rightarrow (0, -1 + \sqrt{13})$

$(0, -1 - \sqrt{13})$

asymptotes $y = k \pm \frac{a}{b}(x - h)$: $y = (-1) \pm \frac{2}{3}(x - 0)$

$y = (-1) \pm \frac{2}{3}x$

$y = (-1) + \frac{2}{3}x$ $\qquad$ $y = (-1) - \frac{2}{3}x$



$9(y + 1)^2 - 4x^2 = 36$

$9(y + 1)^2 = 4x^2 + 36$

$(y + 1)^2 = \frac{4}{9}x^2 + 4$

$y + 1 = \pm \sqrt{\frac{4}{9}x^2 + 4}$

$y = (-1) \pm \sqrt{\frac{4}{9}x^2 + 4}$ **graphing form**

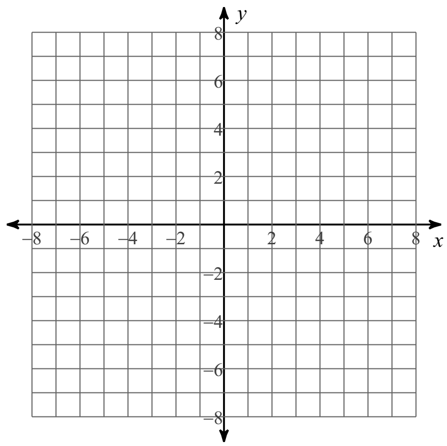
Precalculus Name________________________________

CW26 - Circles

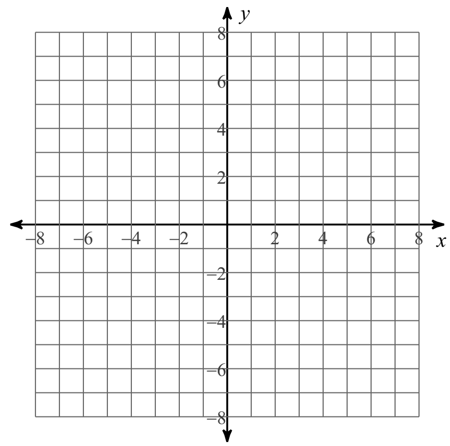
Period____

Identify the center and radius of each. Then sketch the graph.

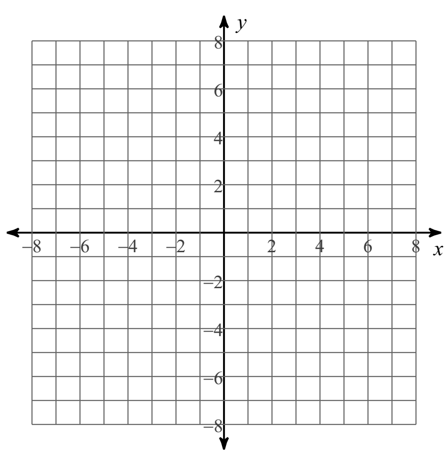
1) $13 + y^2 + x^2 = -8x - 2y$



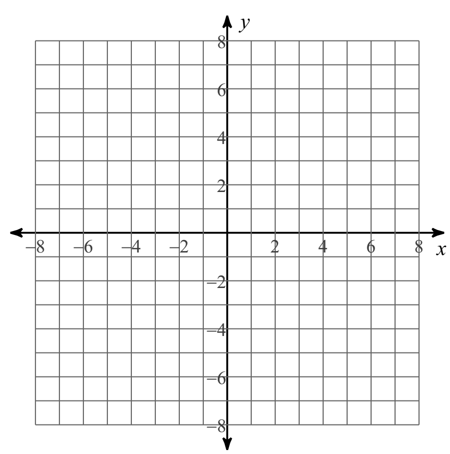
2) $x^2 - 8x + 4y + y^2 = -13$



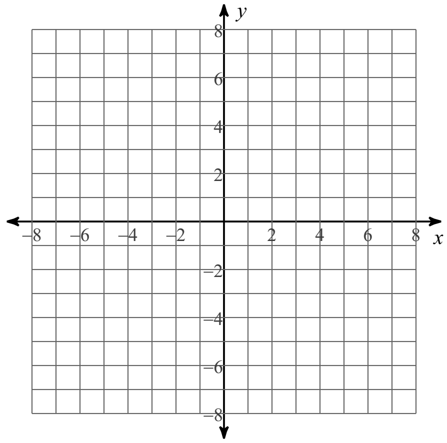
3) $(x + 1)^2 + (y - 4)^2 = 1$



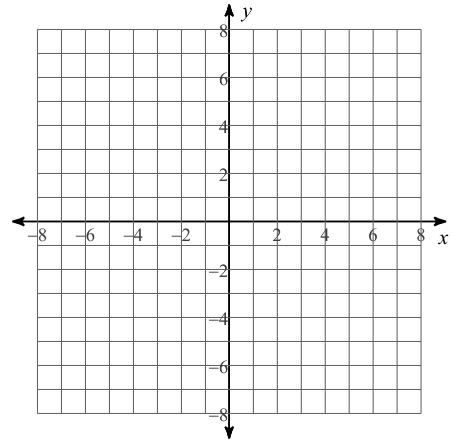
4) $14 + x^2 = -y^2 + 8x + 2y$



5) $\left(x-\frac{3}{2}\right)^2+(y-4)^2=9$



6) $x^2+y^2-6x+4y+9=0$



Use the information provided to write the standard form equation of each circle.

7) Center: $(8, 4)$
Radius: 7

8) Center: $(14, -13)$
Radius: 1

9) Center: $(6, -5)$
Radius: $\sqrt{131}$

10) Center: $\left(-\frac{1}{2}, -\frac{13}{2}\right)$
Radius: 8

Use the information provided to write the general conic form equation of each circle.

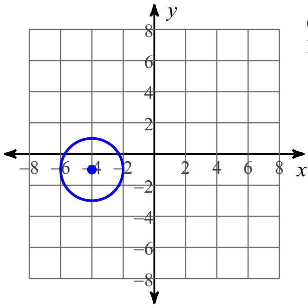
11) Center: $(7, -8)$
Radius: 8

12) Center: $(-13, -4)$
Radius: 5

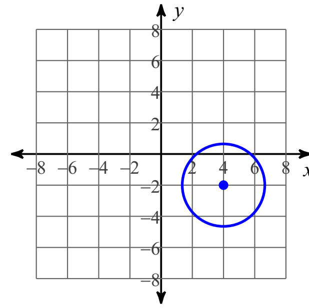
13) Center: $(-8, 1)$
Radius: 10

14) Center: $(10, 6)$
Radius: $\sqrt{67}$

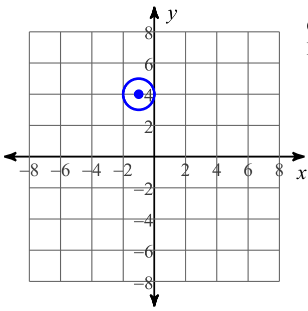
Answers to CW26 - Circles

1) 

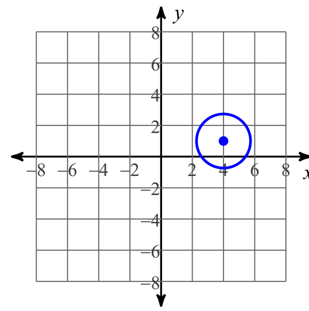
Center: $(-4, -1)$
Radius: 2

2) 

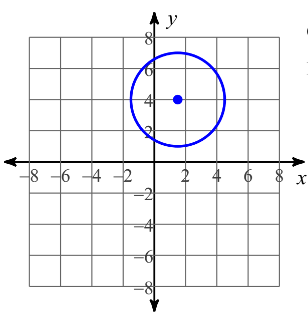
Center: $(4, -2)$
Radius: $\sqrt{7}$

3) 

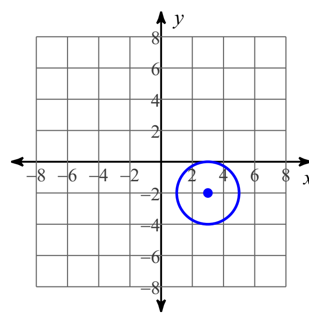
Center: $(-1, 4)$
Radius: 1

4) 

Center: $(4, 1)$
Radius: $\sqrt{3}$

5) 

Center: $\left(\frac{3}{2}, 4\right)$
Radius: 3

6) 

Center: $(3, -2)$
Radius: 2

7) $(x - 8)^2 + (y - 4)^2 = 49$

8) $(x - 14)^2 + (y + 13)^2 = 1$

9) $(x - 6)^2 + (y + 5)^2 = 131$

10) $\left(x + \frac{1}{2}\right)^2 + \left(y + \frac{13}{2}\right)^2 = 64$

11) $x^2 + y^2 - 14x + 16y + 49 = 0$

12) $x^2 + y^2 + 26x + 8y + 160 = 0$

13) $x^2 + y^2 + 16x - 2y - 35 = 0$

14) $x^2 + y^2 - 20x - 12y + 69 = 0$

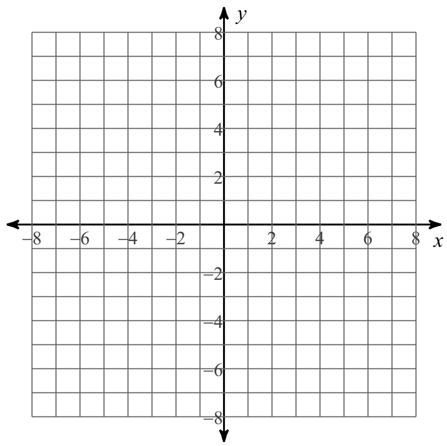
Precalculus Name________________________________

HW26 - Circles

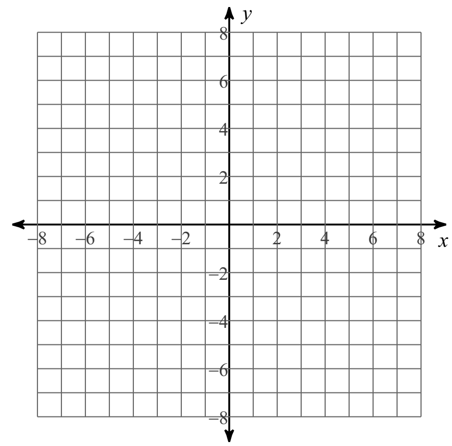
Period____

Identify the center and radius of each. Then sketch the graph.

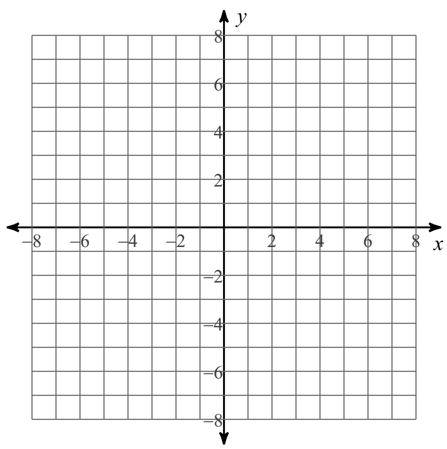
1) $-21 = -x^2 - y^2 + 4y$



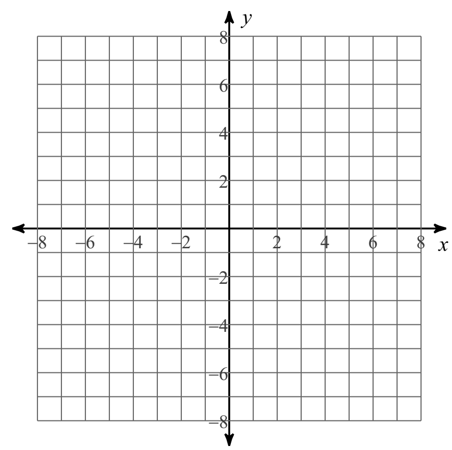
2) $x^2 + y^2 - 2x - 2y - 2 = 0$



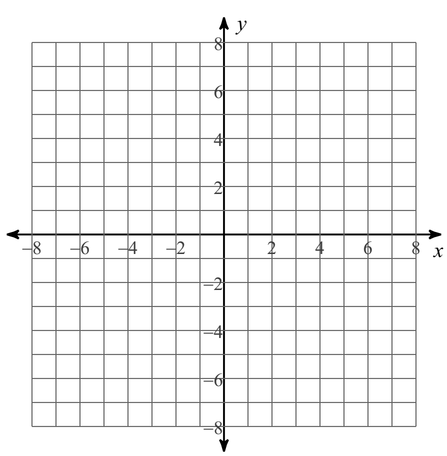
3) $(x - 4)^2 + (y - 4)^2 = 5$



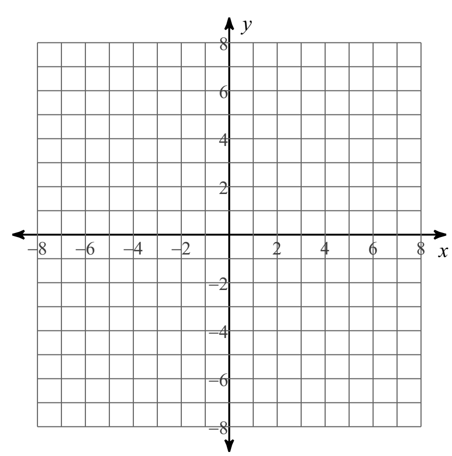
4) $\left(x - \sqrt{5}\right)^2 + \left(y - \frac{5}{2}\right)^2 = 15$



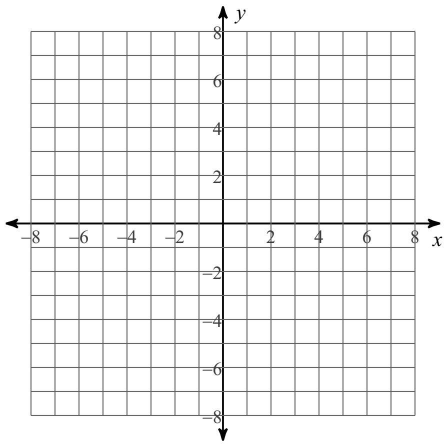
5) $x^2 + 22 - 2y\sqrt{14} + y^2 = 6x$



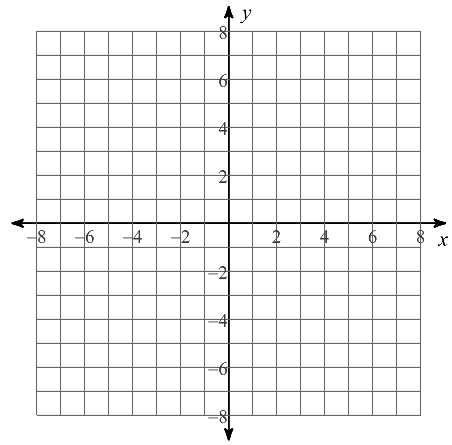
6) $x^2 - 3 - 2x = -2y - y^2$



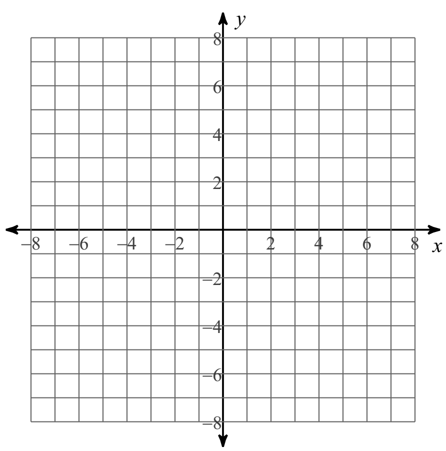
7) $y^2 + x^2 + 6y - 2x = 6$



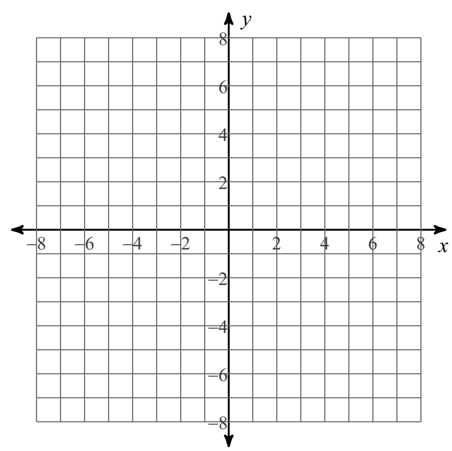
8) $2y + y^2 = 24 - x^2$



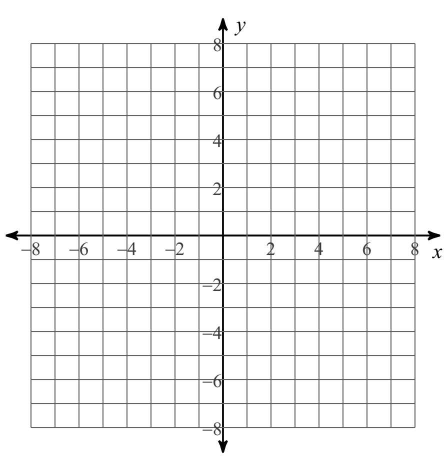
9) $-6x - 4y = -y^2 - x^2 + 3$



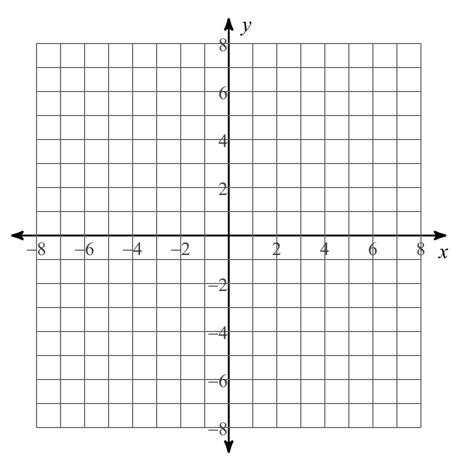
10) $x^2 = -6y - 14 - y^2 - 6x$



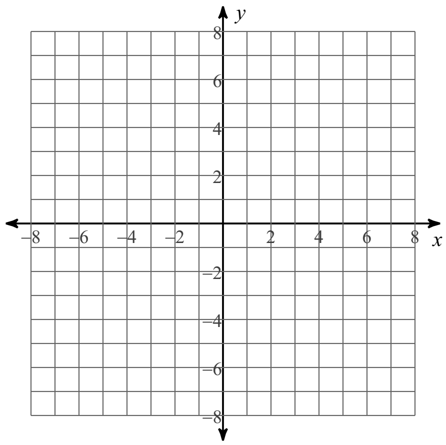
11) $y^2 + 4x = -9 - x^2 + 6y$



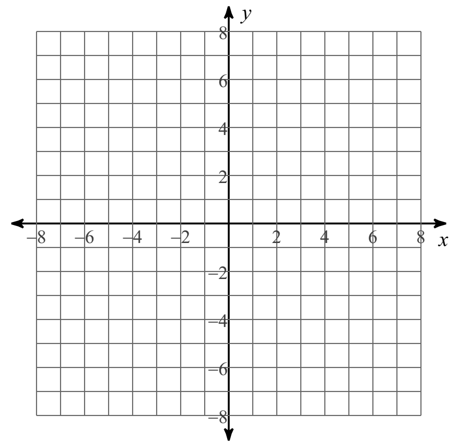
12) $x^2 + y^2 - 4x + 4y - 15 = 0$



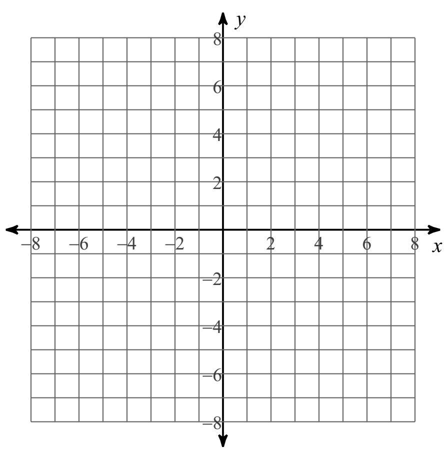
13) $x^2 + y^2 - 4x + 8y + 13 = 0$



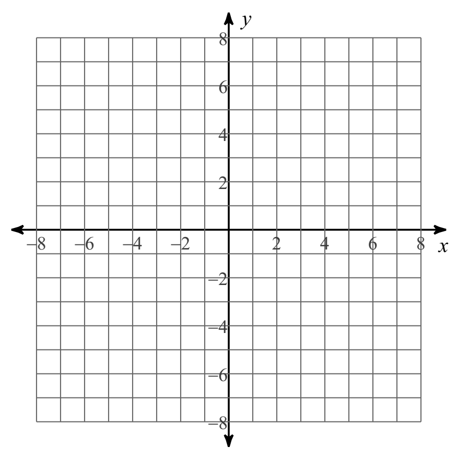
14) $x^2 + y^2 - 8x - 8y + 31 = 0$



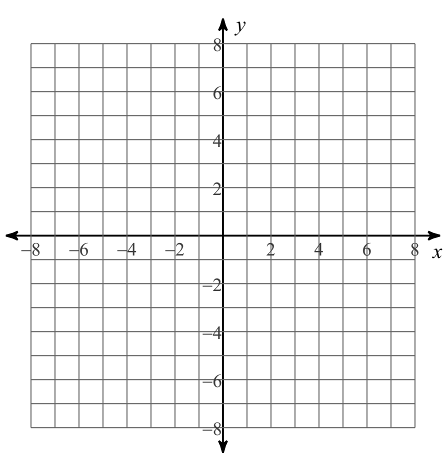
15) $y^2 + x^2 = 6x$



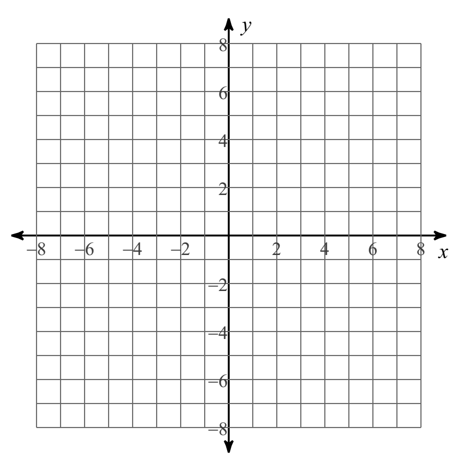
16) $x^2 + 2y = 6x + 6 - y^2$



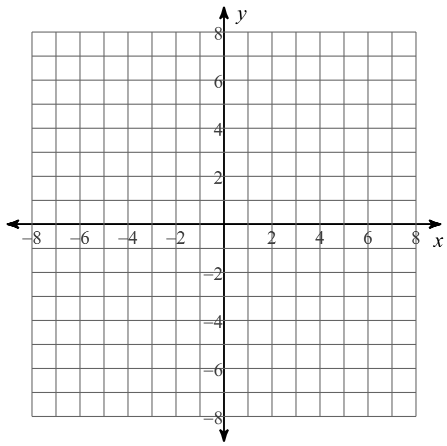
17) $x^2 + (y - 4)^2 = 9$



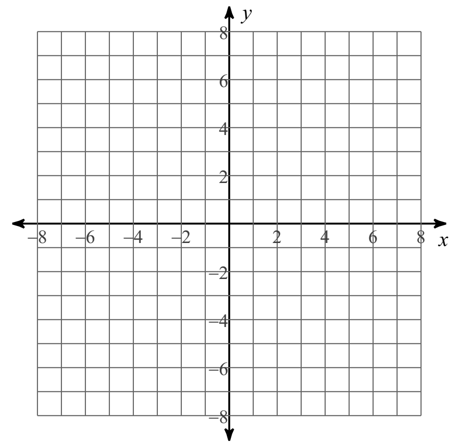
18) $-2y + y^2 = -11 - x^2 - 8x$



19) $y^2 - 1 + 4x = -4y - x^2$



20) $x^2 + y^2 = 16$



Use the information provided to write the standard form equation of each circle.

21) Center: $(0, 4)$
Radius: 7

22) Center: $(-5, 2)$
Radius: $\sqrt{42}$

23) Center: $(12, 14)$
Radius: $\sqrt{5}$

24) Center: $(5, 3)$
Radius: 9

25) Center: $(-7, -9)$
Radius: 5

26) Center: $(-8, 15)$
Radius: 2

27) Center: $(-16, -10)$
Radius: 1

28) Center: $(3, 7)$
Radius: $\sqrt{53}$

29) Center: $(12, 3)$
Radius: 4

30) Center: $(-2, 9)$
Radius: 9

Use the information provided to write the general conic form equation of each circle.

31) Center: $(3, 10)$
Radius: 5

32) Center: $\left(\frac{11}{2}, 6\sqrt{3}\right)$
Radius: $3\sqrt{6}$

33) Center: $(1, 5)$
Radius: 6

34) Center: $(13, 10)$
Radius: 3

35) Center: $(7, 12)$
Radius: 3

36) Center: $(-15, 7)$
Radius: 2

37) Center: $(7, 16)$
Radius: 1

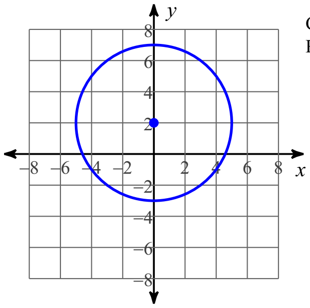
38) Center: $(2, -12)$
Radius: 4

39) Center: $\left(\frac{17}{2}, -\frac{7}{2}\right)$
Radius: 1

40) Center: $\left(\frac{23}{2}, -\frac{17}{2}\right)$
Radius: 7

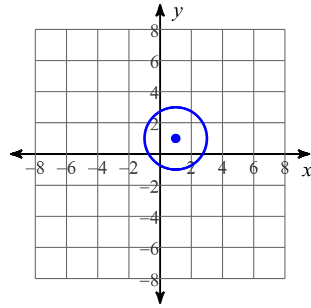
Answers to HW26 - Circles

1)



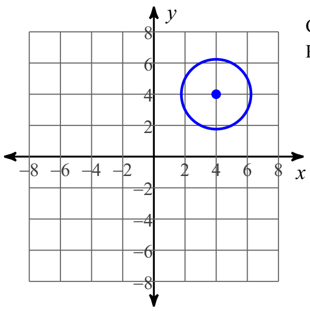
Center: $(0, 2)$
Radius: 5

2)



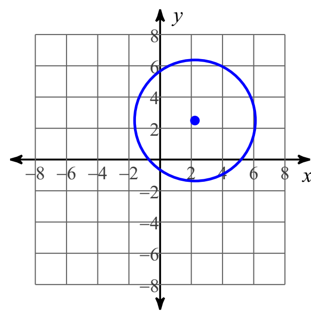
Center: $(1, 1)$
Radius: 2

3)



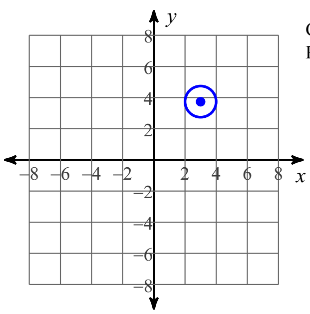
Center: $(4, 4)$
Radius: $\sqrt{5}$

4)



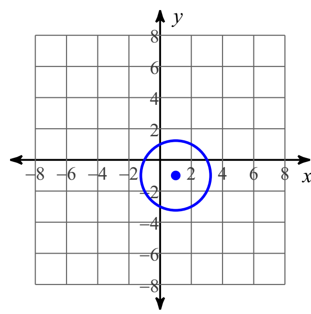
Center: $\left(\sqrt{5}, \frac{5}{2}\right)$
Radius: $\sqrt{15}$

5)



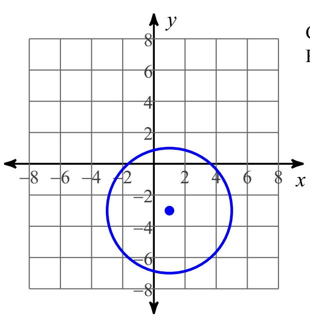
Center: $(3, \sqrt{14})$
Radius: 1

6)



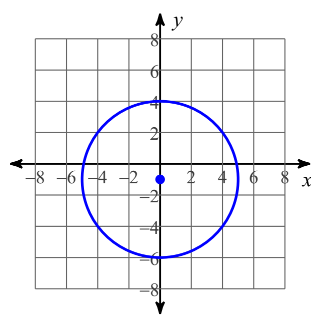
Center: $(1, -1)$
Radius: $\sqrt{5}$

7)



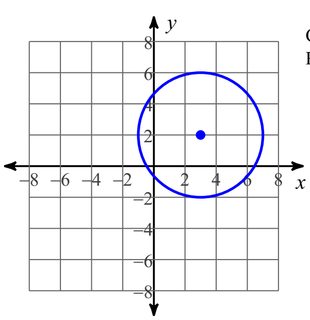
Center: $(1, -3)$
Radius: 4

8)



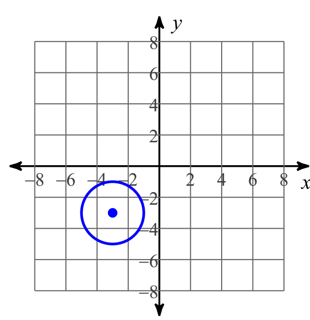
Center: $(0, -1)$
Radius: 5

9)



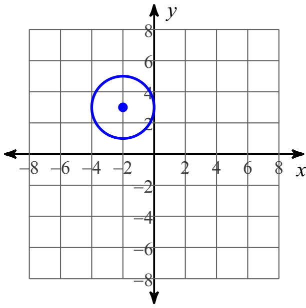
Center: $(3, 2)$
Radius: 4

10)



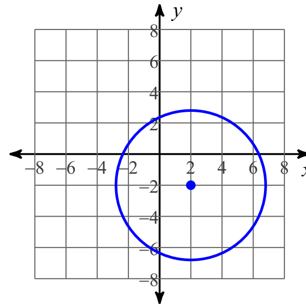
Center: $(-3, -3)$
Radius: 2

11)



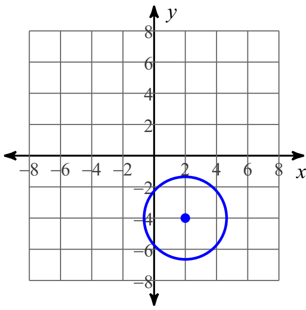
Center: $(-2, 3)$
Radius: 2

12)



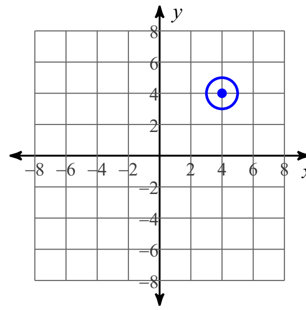
Center: $(2, -2)$
Radius: $\sqrt{23}$

13)



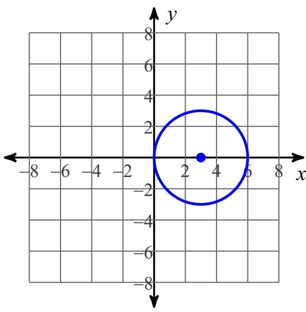
Center: $(2, -4)$
Radius: $\sqrt{7}$

14)



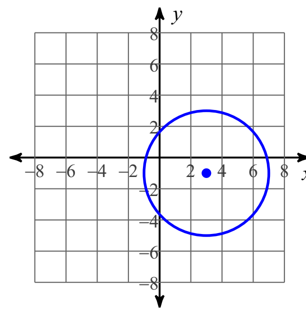
Center: $(4, 4)$
Radius: 1

15)



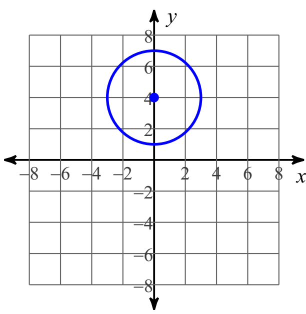
Center: $(3, 0)$
Radius: 3

16)



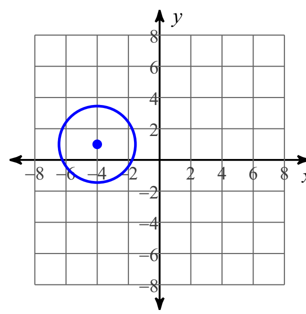
Center: $(3, -1)$
Radius: 4

17)



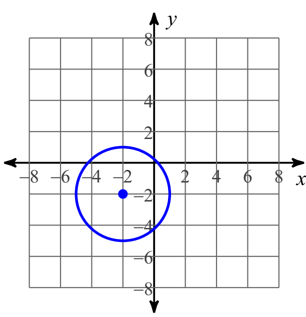
Center: $(0, 4)$
Radius: 3

18)



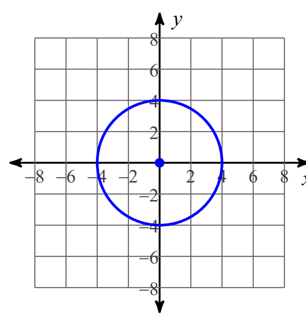
Center: $(-4, 1)$
Radius: $\sqrt{6}$

19)



Center: $(-2, -2)$
Radius: 3

20)



Center: $(0, 0)$
Radius: 4

21) $x^2 + (y - 4)^2 = 49$

22) $(x + 5)^2 + (y - 2)^2 = 42$

23) $(x - 12)^2 + (y - 14)^2 = 5$

24) $(x - 5)^2 + (y - 3)^2 = 81$

25) $(x + 7)^2 + (y + 9)^2 = 25$

26) $(x + 8)^2 + (y - 15)^2 = 4$

27) $(x + 16)^2 + (y + 10)^2 = 1$

28) $(x - 3)^2 + (y - 7)^2 = 53$

29) $(x - 12)^2 + (y - 3)^2 = 16$

30) $(x + 2)^2 + (y - 9)^2 = 81$

31) $x^2 + y^2 - 6x - 20y + 84 = 0$

32) $4x^2 + 4y^2 - 44x - 48y\sqrt{3} + 337 = 0$

33) $x^2 + y^2 - 2x - 10y - 10 = 0$

34) $x^2 + y^2 - 26x - 20y + 260 = 0$

35) $x^2 + y^2 - 14x - 24y + 184 = 0$

36) $x^2 + y^2 + 30x - 14y + 270 = 0$

37) $x^2 + y^2 - 14x - 32y + 304 = 0$

38) $x^2 + y^2 - 4x + 24y + 132 = 0$

39) $2x^2 + 2y^2 - 34x + 14y + 167 = 0$

40) $2x^2 + 2y^2 - 46x + 34y + 311 = 0$