LINES AND AVERAGE RATE OF CHANGE

(x_1, y_1) AND (x_2, y_2) TWO POINTS FORM A LINE

$Ax + By = C$ STANDARD FORM

$y = mx + b$ SLOPE-INTERCEPT FORM

(b: Y-INTERCEPT; m: SLOPE)

SLOPE
- RATE OF CHANGE
- TANGENT
- DERIVATIVE
- GRADIENT
- {RISE OVER RUN}

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\downarrow$$

$y - y_1 = m(x - x_1)$ POINT-SLOPE FORM

OR

$y - y_2 = m(x - x_2)$

PARALLEL LINES
- SAME SLOPE OF THE LINE BEING COMPARED TO

PERPENDICULAR LINES
- OPPOSITE INVERSE OF THE SLOPE OF THE LINE BEING COMPARED TO

$[\ ] \equiv [x_1, x_2]$ RANGE OF X VALUES

$(\ ) \equiv (x_1, y_1)$ POINT CONTAINING X AND Y VALUE

$f(x)$ = polynomial $[x_1, x_2]$ over the interval finding the average rate of change

To find y_1 put x_1 value into the polynomial $y_1 = f(x_1)$

To find y_2 put x_2 value into the polynomial $y_2 = f(x_2)$

EXAMPLES:

GIVEN TWO POINTS FIND THE EQUATION OF THE LINE IN STANDARD FORM.

STEPS:
1. LABLE POINTS
2. SLOPE EQUATION PUT IN VALUES
3. USE THE SLOPE AND A POINT (USE THE FIRST POINT)
4. PUT THESE VALUES INTO $y=mx+b$ SOLVE FOR b (y-intercept)
5. WITH m AND b PUT INTO $y=mx+b$ (which is in the slope-intercept form)
6. REARRANGE THE SLOPE-INTERCEPT EQUATION TO GET THE EQUATION INTO STANDARD FORM $Ax+Bx=C$

$(3,2)$ AND $(-4,-4)$

$x_1\ y_1 \qquad x_2\ y_2$

$$m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}=\frac{(-4)-2}{(-4)-3}=\frac{(-4)+(-2)}{(-4)+(-3)}=\left(\frac{-6}{-7}\right)=\frac{6}{7}$$

$m=\frac{6}{7} \quad (3,2) \quad y=mx+b$

$x\ y$

$$2=\frac{6}{7}(3)+b$$

$$2=\frac{18}{7}+b$$

$$(7)2=\frac{18\cancel{(7)}}{\cancel{7}}+7b$$

$$(\underline{-18})+14=18+7b+(\underline{-18})$$

$$\frac{(-4)}{7}=\frac{7b}{7}$$

$$\left(-\frac{4}{7}\right)=b$$

$m=\frac{6}{7} \quad b=\frac{(-4)}{7} \quad y=mx+b$

$y=\frac{6x}{7}+\left(\frac{-4}{7}\right)$ SLOPE-INTERCEPT FORM

$$7y=(7)\frac{6x}{7}+\left(\frac{-4}{7}\right)7$$

$$(\underline{-6x})+7y=6x+(-4)+(\underline{-6x})$$

$\boxed{(-6x)+7y=(-4)}$ STANDARD FORM

GIVEN TWO POINT FIND THE EQUATION OF A LINE IN SLOPE-INTERCEPT FORM.

STEPS:
FOLLOW STEPS 1-5 LISTED ON THE PREVIOUS PAGE

$(1,-2)$ AND $(-3,2)$
$x_1\ y_1$ $\quad x_2\ y_2$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1} = \frac{2-(-2)}{(-3)-1} = \frac{2+2}{(-3)+(-1)} = \frac{4}{(-4)} = (-1)$$

$m=(-1)$ $\quad (1,-2)$ $\quad y = mx+b$

$$(-2) = (-1)(1)+b$$
$$1+(-2) = (-1)+b\ +1$$
$$(-1) = b$$

$m=(-1)$ $b=(-1)$ $\quad y=mx+b$

$\boxed{y=(-x)+(-1)}$ SLOPE-INTERCEPT FORM

GIVEN TWO POINTS FIND THE EQUATION OF A LINE IN POINT-SLOPE FORM.

STEPS:
FOLLOW STEPS 1 AND 2
3. PICK A POINT TO PUT INTO THE POINT-SLOPE EQUATION $(y-y_1)=m(x-x_1)$ FOR THE FIRST POINT

$(5,1)$ AND $(0,4)$
$x_1\ y_1$ $\quad x_2\ y_2$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2-y_1}{x_2-x_1} = \frac{4-1}{0-5} = \frac{4+(-1)}{(-5)} = \frac{3}{(-5)} = \left(\frac{-3}{5}\right)$$

$m=\left(\frac{-3}{5}\right)$ $\quad (5,1)$ $\quad y-y_1 = m(x-x_1)$

$\boxed{y-1=\left(\frac{-3}{5}\right)(x-5)}$ POINT-SLOPE FORM

WRITE THE SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE THROUGH $(4,2)$ PARALLEL TO $y=(-5x)-1$

$m=(-5)$
$m_{\parallel}=(-5)$ SAME SLOPE

$m_{\parallel}=(-5)$ $\quad (4,2)$ $\quad y=mx+b$

$$2=(-5)4+b$$
$$20+2=(-20)+b\ +20$$
$$22=b$$ put into $y=mx+b$

$\boxed{y=(-5x)+22}$ parallel line to $y=(-5x)+(-1)$ at $(4,2)$

WRITE THE SLOPE-INTERCEPT FORM OF THE EQUATION OF A LINE THROUGH $(3,-5)$ PERPENDICULAR TO $y = \frac{3}{5}x + 1$

$m = \frac{3}{5}$

$m_{\perp} = \left(-\frac{5}{3}\right)$ take the opposite inverse of the original slope

$m_{\perp} = \left(-\frac{5}{3}\right)$ $\underset{x\ \ y}{(3,-5)}$

$y = mx + b$

$(-5) = \left(-\frac{5}{\cancel{3}}\right)\cancel{3} + b$

$5 + (-5) = (-5) + b + 5$

$0 = b$

$m_{\perp} = \left(-\frac{5}{3}\right)$ $b = 0$ $y = mx + b$

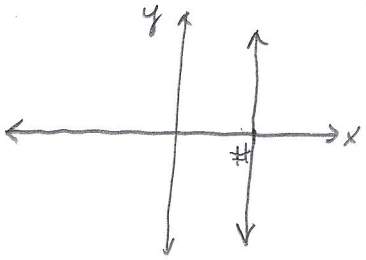
$\boxed{y = \left(-\frac{5x}{3}\right)}$ perpendicular line to $y = \frac{3x}{5} + 1$ at $(3,-5)$

TWO SPECIAL CASES:

1) m IS UNDEFINED $x = \#$

THIS MEANS THAT BOTH X_1 AND X_2 ARE EQUAL

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_2 - y_1}{0}$ UNDEFINED! denominator is zero therefore m IS UNDEFINED

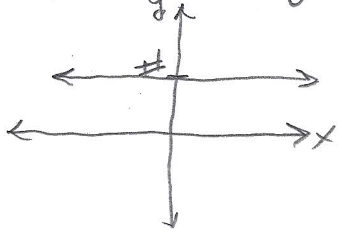


2) $m = 0$ $y = \#$

USUALLY FOR A LINE $y = b$

THIS MEANS THAT BOTH Y_1 AND Y_2 ARE THE SAME

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0}{x_2 - x_1} = 0$



FIND THE AVERAGE RATE OF CHANGE OF THE FUNCTION OVER A GIVEN INTERVAL.

STEPS:
1. LABLE THE INTERVAL
2. SOLVE FOR THE CORRESPONDING Y VALUES FROM THE TWO X VALUES

$f(x) = x^2 + 2x + 4 \quad [\underset{x_1}{-1}, \underset{x_2}{1}]$

$x_1 = (-1)$

$y_1 = f(-1) = (-1)^2 + 2(-1) + 4$

$y_1 = 1 + (-2) + 4$

$y_1 = 3$

$(\underset{x_1}{-1}, \underset{y_1}{3})$

$x_2 = 1$

$y_2 = f(1) = 1^2 + 2(1) + 4$

$y_2 = 1 + 2 + 4$

$y_2 = 7$

$(\underset{x_2}{1}, \underset{y_2}{7})$

3. USING THESE TWO POINTS FIND THE AVERAGE RATE OF CHANGE (SLOPE)

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-3}{1-(-1)} = \frac{7+(-3)}{1+1} = \frac{4}{2} = 2$$

$m = 2$ AVERAGE RATE OF CHANGE

Precalculus Name________________________

CW04 - Average Rates of Change and Secant Lines

Period____

For each problem, find the average rate of change of the function over the given interval.

1) $f(x) = 2x^2 + 2x + 2;\ [-2, -\frac{3}{2}]$

2) $f(x) = x^2 + 2x - 1;\ [-2, -\frac{3}{2}]$

3) $f(x) = 2x^2 - 2;\ [0, \frac{1}{3}]$

4) $f(x) = 2x^2 + 2;\ [-1, -\frac{1}{2}]$

5) $f(x) = -2x^2 + 2;\ [-1, -\frac{3}{4}]$

6) $f(x) = x^2 - x + 2;\ [2, \frac{9}{4}]$

7) $f(x) = x^2 + 2x + 2;\ [-2, -\frac{5}{3}]$

8) $f(x) = x^2 - 2;\ [2, \frac{5}{2}]$

9) $f(x) = x^2 - 1;\ [2, \frac{7}{3}]$

10) $f(x) = 2x^2 + 2x + 1;\ [-1, -\frac{2}{3}]$

11) $f(x) = x^2 - 2;\ [-2, -\frac{7}{4}]$

12) $f(x) = x^2 - 2x + 2;\ [-1, -\frac{2}{3}]$

13) $f(x) = x^2 + x + 2;\ [-2, -\frac{3}{2}]$

14) $f(x) = 2x^2 + 2x + 2;\ [0, \frac{1}{4}]$

15) $f(x) = x^2 + 2x + 1;\ [-1, -\frac{2}{3}]$

16) $f(x) = x^2 + 2x + 1;\ [-2, -\frac{5}{3}]$

17) $f(x) = 2x^2 + x - 1;\ [1, \frac{4}{3}]$

18) $f(x) = -2x^2 + x + 2;\ [-1, -\frac{1}{2}]$

19) $f(x) = x^2 + 2;\ [2, \frac{9}{4}]$

20) $f(x) = x^2 + 2;\ [0, \frac{1}{4}]$

For each problem, find the equation of the secant line that intersects the given points on the function.

21) $f(x) = x^2 + 2x + 2;\ (-1, 1), \left(-\frac{3}{4}, \frac{17}{16}\right)$

22) $f(x) = x^2 + 2x + 2;\ (-1, 1), \left(-\frac{2}{3}, \frac{10}{9}\right)$

23) $f(x) = -2x^2 + 1;\ (-2, -7), \left(-\frac{5}{3}, -\frac{41}{9}\right)$

24) $f(x) = x^2 + 2x - 1;\ (-4, 7), \left(-\frac{7}{2}, \frac{17}{4}\right)$

25) $f(x) = 2x^2 + x + 1;\ (0, 1), \left(\frac{1}{3}, \frac{14}{9}\right)$

Answers to CW04 - Average Rates of Change and Secant Lines

1) -5

2) $-\frac{3}{2}$

3) $\frac{2}{3}$

4) -3

5) $\frac{7}{2}$

6) $\frac{13}{4}$

7) $-\frac{5}{3}$

8) $\frac{9}{2}$

9) $\frac{13}{3}$

10) $-\frac{4}{3}$

11) $-\frac{15}{4}$

12) $-\frac{11}{3}$

13) $-\frac{5}{2}$

14) $\frac{5}{2}$

15) $\frac{1}{3}$

16) $-\frac{5}{3}$

17) $\frac{17}{3}$

18) 4

19) $\frac{17}{4}$

20) $\frac{1}{4}$

21) $y = \frac{1}{4}x + \frac{5}{4}$

22) $y = \frac{1}{3}x + \frac{4}{3}$

23) $y = \frac{22}{3}x + \frac{23}{3}$

24) $y = -\frac{11}{2}x - 15$

25) $y = \frac{5}{3}x + 1$

Precalculus Name________________________________

HW04 - Average Rates of Change and Secant Lines

Period____

For each problem, find the average rate of change of the function over the given interval.

1) $f(x) = x^2 - 2$; $[-2, -\frac{7}{4}]$

2) $f(x) = 2x^2 + 2x - 2$; $[-2, -\frac{7}{4}]$

3) $f(x) = x^2 + x + 1$; $[0, \frac{1}{4}]$

4) $f(x) = x^2 + 1$; $[2, \frac{7}{3}]$

5) $f(x) = -x^2 + 2$; $[0, \frac{1}{4}]$

6) $f(x) = x^2 + 2$; $[-1, -\frac{3}{4}]$

7) $f(x) = x^2 + 2x + 1$; $[0, \frac{1}{3}]$

8) $f(x) = 2x^2 + 2$; $[-1, -\frac{3}{4}]$

9) $f(x) = x^2 - x + 1$; $[-2, -\frac{5}{3}]$

10) $f(x) = x^2 + 2$; $[1, \frac{4}{3}]$

11) $f(x) = 2x^2 + x + 1$; $[1, \frac{5}{4}]$

12) $f(x) = x^2 + 1$; $[-2, -\frac{3}{2}]$

13) $f(x) = 2x^2 + 1$; $[1, \frac{3}{2}]$

14) $f(x) = 2x^2 + x + 2$; $[-1, -\frac{1}{2}]$

15) $f(x) = -x^2 + x + 1$; $[0, \frac{1}{2}]$

16) $f(x) = 2x^2 + 2x + 1$; $[-1, -\frac{2}{3}]$

17) $f(x) = x^2 + 2$; $[0, \frac{1}{3}]$

18) $f(x) = x^2 - x + 2$; $[-1, -\frac{1}{2}]$

19) $f(x) = 2x^2 + 2x + 1$; $[1, \frac{5}{4}]$

20) $f(x) = -x^2 + 2$; $[-1, -\frac{1}{2}]$

21) $f(x) = 2x^2 + 1$; $[-1, -\frac{3}{4}]$

22) $f(x) = x^2 + 2x + 2$; $[-1, -\frac{2}{3}]$

23) $f(x) = 2x^2 - 2x + 1$; $[1, \frac{5}{4}]$

24) $f(x) = 2x^2 - 2x + 1$; $[-1, -\frac{2}{3}]$

25) $f(x) = -x^2 + 2x + 1$; $[1, \frac{4}{3}]$

26) $f(x) = 2x^2 + x + 1$; $[0, \frac{1}{2}]$

27) $f(x) = x^2 + x - 2$; $[0, \frac{1}{4}]$

28) $f(x) = -2x^2 + 2x + 1$; $[0, \frac{1}{3}]$

29) $f(x) = x^2 + x - 2$; $[-3, -\frac{5}{2}]$

30) $f(x) = 2x^2 - 2$; $[0, \frac{1}{4}]$

31) $f(x) = x^2 + x - 1;\ \ [-2, -\frac{5}{3}]$

32) $f(x) = 2x^2 + x + 1;\ \ [1, \frac{3}{2}]$

33) $f(x) = x^2 + 2;\ \ [1, \frac{5}{4}]$

34) $f(x) = -2x^2 + 2x + 1;\ \ [-1, -\frac{1}{2}]$

35) $f(x) = x^2 - 2;\ \ [2, \frac{9}{4}]$

36) $f(x) = x^2 + 1;\ \ [-2, -\frac{7}{4}]$

37) $f(x) = x^2 - 2;\ \ [-3, -\frac{11}{4}]$

38) $f(x) = -x^2 + x + 2;\ \ [3, \frac{7}{2}]$

39) $f(x) = x^2 + 1;\ \ [0, \frac{1}{2}]$

40) $f(x) = x^2 + 2x + 1;\ \ [1, \frac{4}{3}]$

41) $f(x) = 2x^2 + 2;\ \ [1, \frac{3}{2}]$

42) $f(x) = x^2 + x + 1;\ \ [0, \frac{1}{2}]$

43) $f(x) = x^2 + 2x + 1;\ \ [0, \frac{1}{2}]$

44) $f(x) = 2x^2 - x + 2;\ \ [0, \frac{1}{2}]$

45) $f(x) = -2x^2 + 1;\ \ [1, \frac{4}{3}]$

46) $f(x) = 2x^2 + 2;\ \ [0, \frac{1}{3}]$

47) $f(x) = 2x^2 + 1;\ \ [0, \frac{1}{4}]$

48) $f(x) = x^2 - x + 1;\ \ [-1, -\frac{3}{4}]$

49) $f(x) = x^2 + 1;\ \ [2, \frac{9}{4}]$

50) $f(x) = 2x^2 - x + 2;\ \ [1, \frac{4}{3}]$

For each problem, find the equation of the secant line that intersects the given points on the function.

51) $f(x) = -2x^2 + x - 1;\ (1, -2), \left(\frac{4}{3}, -\frac{29}{9}\right)$

52) $f(x) = -x^2 + 2x - 1;\ (1, 0), \left(\frac{4}{3}, -\frac{1}{9}\right)$

53) $f(x) = -2x^2 + 1;\ (-1, -1), \left(-\frac{2}{3}, \frac{1}{9}\right)$

54) $f(x) = -x^2 + x + 1;\ (1, 1), \left(\frac{3}{2}, \frac{1}{4}\right)$

55) $f(x) = -x^2 + 2;\ (-2, -2), \left(-\frac{7}{4}, -\frac{17}{16}\right)$

56) $f(x) = -2x^2 + x + 1;\ (1, 0), \left(\frac{3}{2}, -2\right)$

57) $f(x) = x^2 - 2x + 1;\ (0, 1), \left(\frac{1}{4}, \frac{9}{16}\right)$

58) $f(x) = -x^2 - x + 1;\ (-1, 1), \left(-\frac{2}{3}, \frac{11}{9}\right)$

59) $f(x) = x^2 + x - 2;\ (-3, 4), \left(-\frac{5}{2}, \frac{7}{4}\right)$

60) $f(x) = 2x^2 + x + 2;\ (-1, 3), \left(-\frac{3}{4}, \frac{19}{8}\right)$

Answers to HW04 - Average Rates of Change and Secant Lines

1) $-\frac{15}{4}$
2) $-\frac{11}{2}$
3) $\frac{5}{4}$
4) $\frac{13}{3}$
5) $-\frac{1}{4}$
6) $-\frac{7}{4}$
7) $\frac{7}{3}$
8) $-\frac{7}{2}$
9) $-\frac{14}{3}$
10) $\frac{7}{3}$
11) $\frac{11}{2}$
12) $-\frac{7}{2}$
13) 5
14) -2
15) $\frac{1}{2}$
16) $-\frac{4}{3}$
17) $\frac{1}{3}$
18) $-\frac{5}{2}$
19) $\frac{13}{2}$
20) $\frac{3}{2}$
21) $-\frac{7}{2}$
22) $\frac{1}{3}$
23) $\frac{5}{2}$
24) $-\frac{16}{3}$
25) $-\frac{1}{3}$
26) 2
27) $\frac{5}{4}$
28) $\frac{4}{3}$
29) $-\frac{9}{2}$
30) $\frac{1}{2}$
31) $-\frac{8}{3}$
32) 6
33) $\frac{9}{4}$
34) 5
35) $\frac{17}{4}$
36) $-\frac{15}{4}$
37) $-\frac{23}{4}$
38) $-\frac{11}{2}$
39) $\frac{1}{2}$
40) $\frac{13}{3}$
41) 5
42) $\frac{3}{2}$
43) $\frac{5}{2}$
44) 0
45) $-\frac{14}{3}$
46) $\frac{2}{3}$
47) $\frac{1}{2}$
48) $-\frac{11}{4}$
49) $\frac{17}{4}$
50) $\frac{11}{3}$
51) $y=-\frac{11}{3}x+\frac{5}{3}$
52) $y=-\frac{1}{3}x+\frac{1}{3}$
53) $y=\frac{10}{3}x+\frac{7}{3}$
54) $y=-\frac{3}{2}x+\frac{5}{2}$
55) $y=\frac{15}{4}x+\frac{11}{2}$
56) $y=-4x+4$
57) $y=-\frac{7}{4}x+1$
58) $y=\frac{2}{3}x+\frac{5}{3}$
59) $y=-\frac{9}{2}x-\frac{19}{2}$
60) $y=-\frac{5}{2}x+\frac{1}{2}$